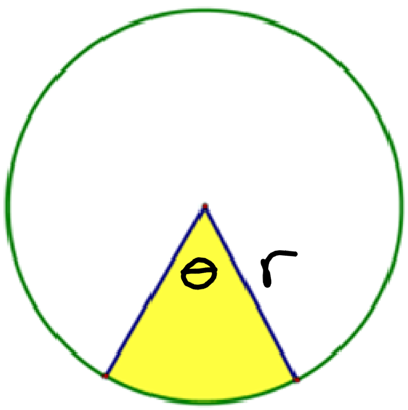
10-3 day 2 Calculus of Polar Functions

Learning Objectives:

I can find the area enclosed by a polar curve

I can find the area between two polar curves



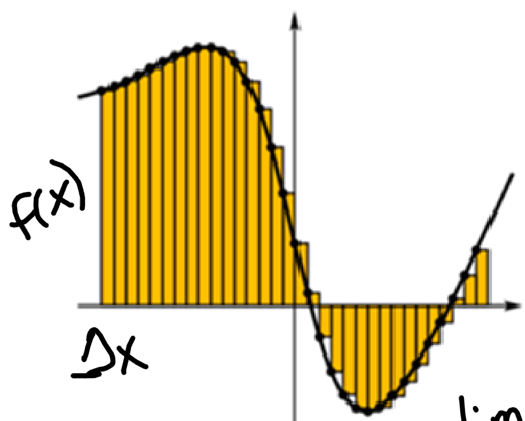
Area of a Sector

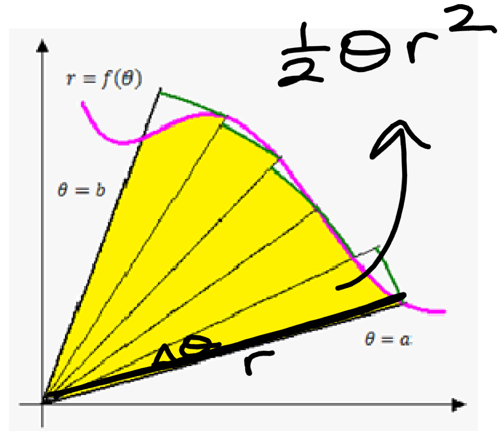
$$A(\bigcirc) = \pi r^2$$

$$A(\triangledown) = \frac{\theta}{2\pi} \cdot \pi r^2$$

$$\boxed{A = \frac{1}{2}\theta r^2}$$

Rectangular Coordinates	Polar Coordinates



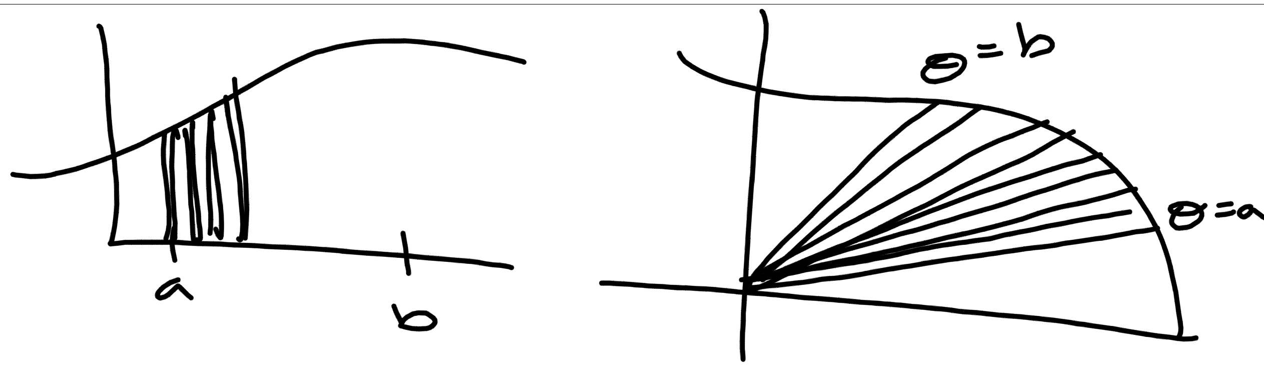


$$\sum f(x)\cdot\Delta x$$

$$\int f(x)\,dx$$

$$\lim_{\Delta x\to 0}\sum \frac{1}{2}r^2\Delta\theta$$

$$\int \frac{1}{2}r^2\,d\theta$$



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Area in Polar Coordinates

The area of the region between the origin and the curve is $r = f(\theta)$

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta = \int_{\alpha}^{\beta} \frac{1}{2} \left(f(\theta) \right)^2 d\theta$$

Where the curve is swept out exactly once as $\alpha \leq \theta \leq \beta$

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Ex1. Find the area in the plane enclosed inside the limacon $r = 3 + 2\sin(\theta)$

$$A = \int_0^{2\pi} \frac{1}{2}(3+2\sin\theta)^2 =$$

$$34.558$$

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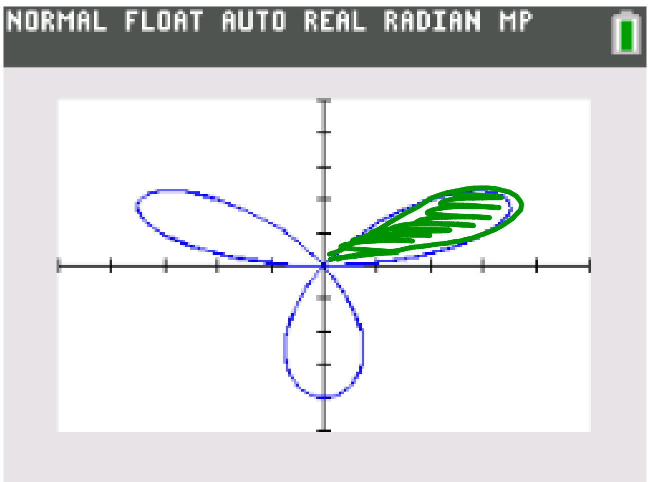
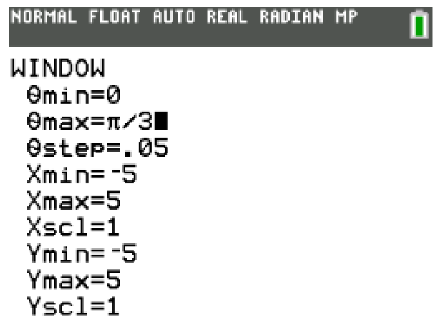
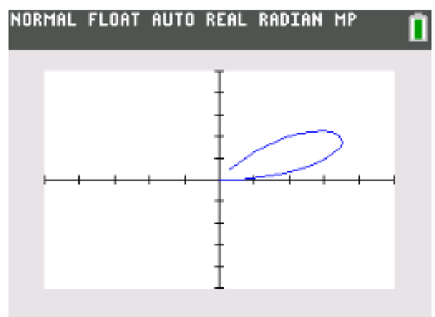
Ex2. Find the area enclosed inside the petal in the first quadrant of the rose $r = 4\sin(3\theta)$



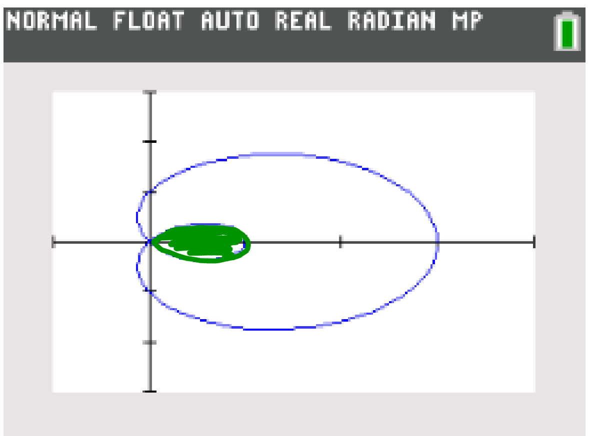




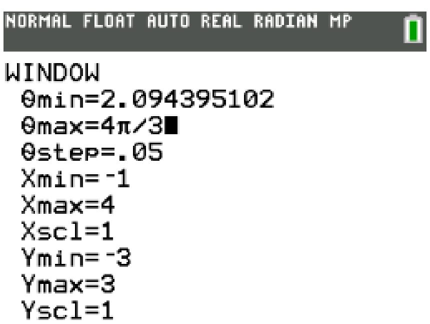
$$\int_0^{\pi/3} \frac{1}{2}\left(4\sin(3\theta)\right)^2 d\theta$$

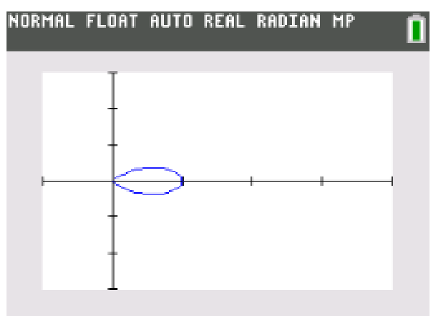
$$\approx 4.189$$

Ex3. Find the area enclosed inside the small loop of the limacon $r = 1 + 2\cos(\theta)$

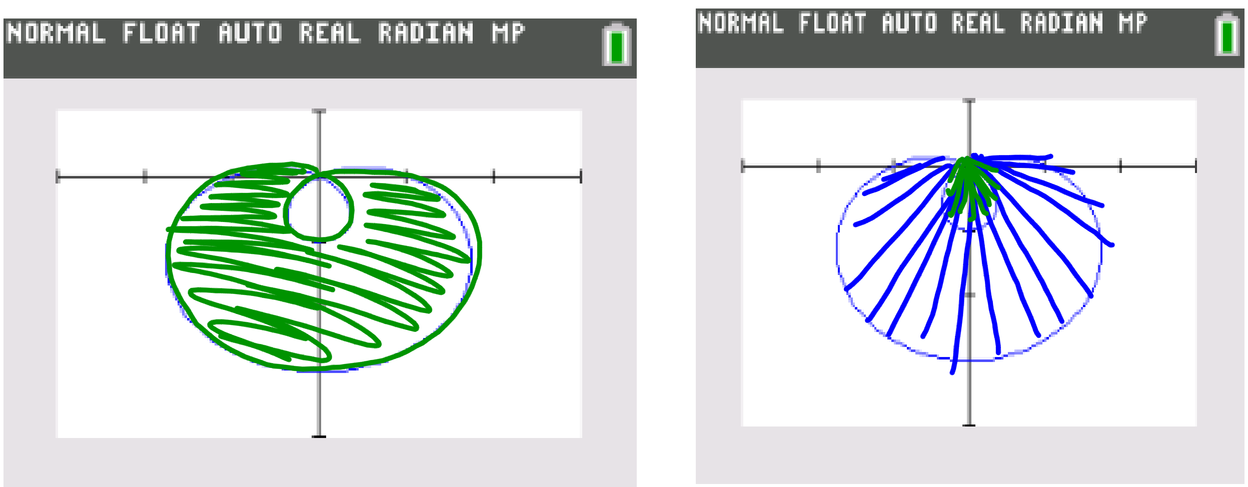


$$\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left(\frac{1}{2}(1+2\cos\theta)^2\right) d\theta \approx .544$$





Ex4. Find the area lying between the inner and outer loops of the limacon $r = 1 - 2\sin(\theta)$



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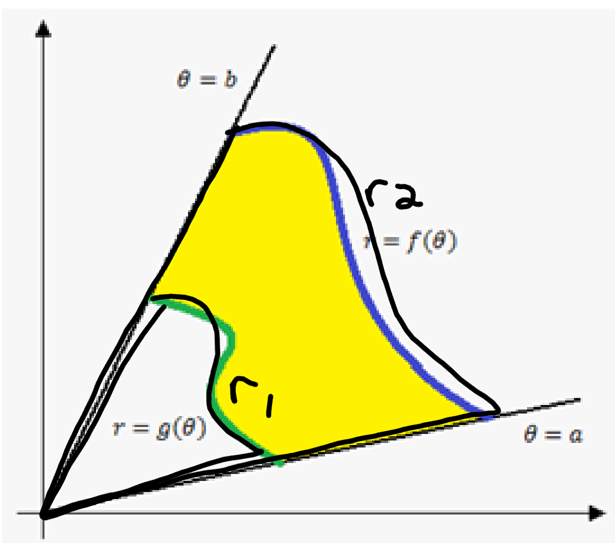
Area between two Polar Coordinates

Let the Area A be the area bounded by the curves $r_1 = g(\theta)$, $r_2 = f(\theta)$, $\theta = \alpha$, $\theta = \beta$. Then the area of A is given by:



$$A = \int_\alpha^\beta \frac{1}{2} r_2^{\,2}\, d\theta - \int_\alpha^\beta \frac{1}{2} r_1^{\,2}\, d\theta$$

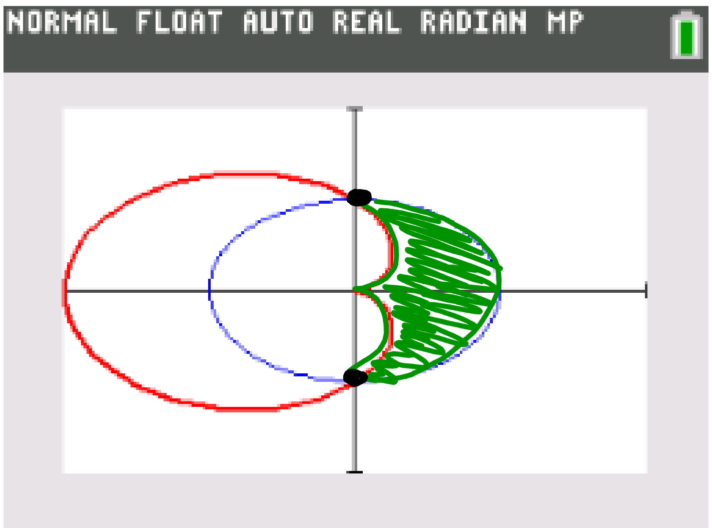
$$A = \int_\alpha^\beta \frac{1}{2}\left(r_2^{\,2} - r_1^{\,2}\right) d\theta$$

$$A = \int_\alpha^\beta \frac{1}{2}\left(\left(f(\theta)\right)^2 - \left(g(\theta)\right)^2\right) d\theta$$

$$A = \int_\alpha^\beta \frac{1}{2} r_o^{\,2} - \frac{1}{2} r_i^{\,2}\, d\theta$$

$$A = \frac{1}{2}\int_\alpha^\beta r_o^{\,2} - r_i^{\,2}\, d\theta$$

Ex5. Find the area of the region that lies inside the circle r=1 and outside the cardioid $r = 1 - \cos(\theta)$



$$A = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1-(1-\cos\theta)^2 \, d\theta$$

$$\boxed{\approx 1.215}$$

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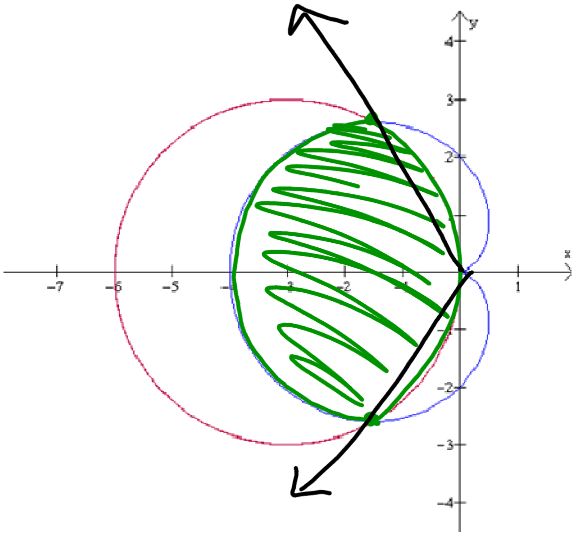
Ex6. Find the shaded area

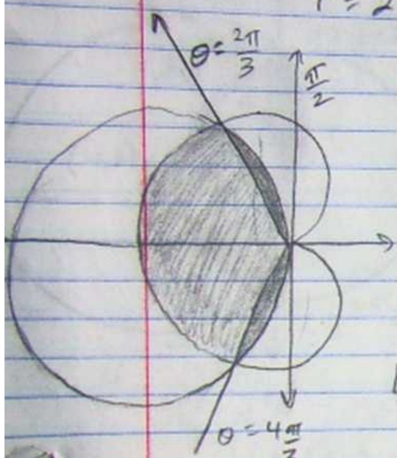


$r = -6\cos(\theta)$

$r = 2 - 2\cos(\theta)$

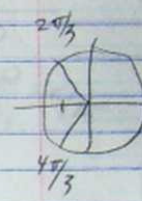
$r = -6\cos\theta$ (circle)

$r = 2 - 2\cos\theta$ (cardioid)



pts of intersection

$$-6\cos\theta = 2 - 2\cos\theta$$
$$+2\cos\theta \qquad +2\cos\theta$$
$$\frac{-4\cos\theta}{-4} = \frac{2}{-4}$$
$$\cos\theta = -1/2$$
$$\theta = 2\pi/3,\ 4\pi/3$$



Lets just work with the upper 1/2 and double our answer.

$$A = \int_{\pi/2}^{2\pi/3} \frac{1}{2}(-6\cos\theta)^2\,d\theta + \int_{2\pi/3}^{\pi} \frac{1}{2}(2-2\cos\theta)^2\,d\theta$$

$$\approx .8153 + 7.0387 \approx 7.854\,u^2$$

$$\times 2 = \boxed{15.708\,u^2}$$

OR

$$9\pi - \int_{2\pi/3}^{4\pi/3} \frac{1}{2}\left((2-2\cos\theta)^2 - (-6\cos\theta)^2\right)d\theta$$

$$\approx 15.708\,u^2$$

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Homework

pg 558 #44-56 even, 57-60, 62-64